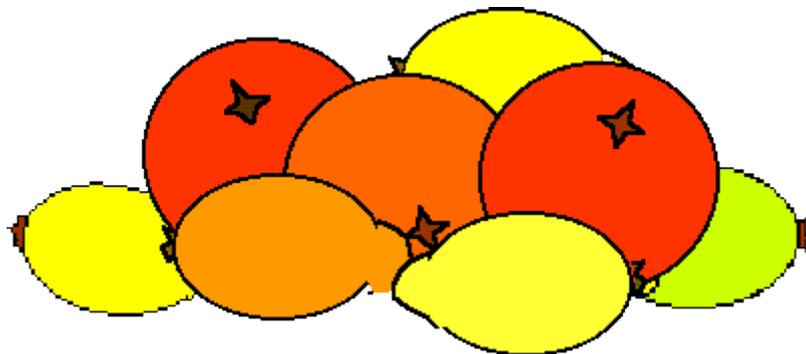


Oranges and Lemons

On the table there is a pile of oranges and lemons that weighs exactly one kilogram.



The oranges all weigh 130 grams. The lemons are also all the same weight, which is less than $\frac{2}{3}$ of the weight of an orange.

There are twice as many lemons as oranges in the pile.

How many lemons are there and how much does each one weigh?

THINK ABOUT:

What is the largest number of oranges you can have and their total weight be less than (or equal to) one kilogram?

If you have one orange how many lemons would there be?

How much would the orange weigh and what is the most the two lemons can weigh?

Key questions

What is the most a lemon can weigh?

Do you have to have a multiple of 3 lemons?

Is there more than one possible answer?

How do you know you have all the solutions?

SOLUTION

Three girls explain it as follows: We think we have the answer:

We think that the answer is 8 lemons, which have a mass of 60g,

and 4 oranges.

What strategy did the girls use to arrive at that answer? They explain:

We got the answer by trial and improvement.

That last point is very important! All the people who wrote in agree with the girls. But Luke shows his calculations to prove to us that there could be more than one possible answer. Do you agree with Luke?

Oranges and Lemons Luke Catchpole
(aged 10)

A There is 8 lemons and each one weighs 60g.

$$\begin{array}{r} 1000 \\ - 520 \quad 4 \times 130g \text{ oranges} \\ \hline 480 \\ \div 60 \\ \hline 8 \quad \text{twice as many lemons} \end{array}$$

B There are 10 lemons and each one weighs 35g.

$$\begin{array}{r} 1000 \\ - 650 \quad 5 \times 130g \text{ oranges} \\ \hline 350 \\ \div 35 \\ \hline 10 \quad \text{twice as many lemons} \end{array}$$

Pupils at Alice Smith International School in Kuala Lumpur, Malaysia also worked on the Oranges and Lemons problem. They agree with Luke. One of the pupils, Kevin, shows all of the possibilities. They prove that there are in fact two possibilities but that there are no more.

What they already know:

Weights = 1 orange weighs 130g. 1 lemon weighs less than 87g.

To make it easy to read the answers, the pupils use a table:

Number of oranges **Number of lemons** **Right / Wrong**

1	2	Wrong
2	4	Wrong
3	6	Wrong
4	8	Right
5	10	Right
6	12	Wrong
7	14	Wrong
8	16	Wrong

To show how they arrived at each of these answers, here are the calculations:

1. $130g + (87g \times 2)$ 87g is max weight for Lemons = 304g
2. $(130g \times 2) + (87g \times 4) = 608g$
3. $(130g \times 3) + (87g \times 6) = 912g$
4. $(130g \times 4) + (60g \times 8) = 1000g$ or 1kg
5. $(130g \times 5) + (35g \times 10) = 1000g$ or 1kg
6. $(130g \times 6) = 780g$. $1000g - 780g = 220g$. 220g divided by 12 is 18.3 recurring. It isn't a whole number.
7. $130g \times 7 = 910g$. $1000g - 910g = 90g$. 90 divided by 14 equals 6.428571429. It isn't a whole number.
8. $130g \times 8 = 1040g$. It is more than 1000g or 1kg